Lecture 5, Sep 21, 2021

How do we know how big forces really are compared to the element? Is a load of 100MN in a bridge cable a lot of force? We can use *stress* and *strain*, which are normalized concepts of forces, so we can compare them for elements of different sizes.

Hooke's Law

- The restoring force in a spring is proportional to its change in length: $F = k\Delta t$
- Hooke's law also applies to structures subjected to direct tension or compression such as cables and columns
- Structures that obey Hooke's law are **linear elastic**
- k is sometimes refired to as the *axial stiffness*

Stress and Strain

- *Stress* is an area-normalized measurement of internal force; for a cable carrying a force F with undeformed cross-sectional area A, the stress is $\sigma = \dfrac{F}{A}$
  - Stress has units of force per unit area and is usually in MPa ($1\text{Pa} = 1\text{N}/\text{m}^2$; $1\text{MPa} = \dfrac{1\text{MN}}{\text{m}^2} = \dfrac{1\text{N}}{\text{mm}^2}$)
  - Stress is similar to pressure, but pressure acts externally while stress acts internally
- *Strain* is a length-normalized measurement of deformation; if a cable was originally L_0 units long but has now been stretched by an additional Δl, the strain is $\varepsilon = \dfrac{\Delta l}{L_0}$
  - Strain is dimensionless but typically presented in units of mm/mm, mm/m or even percentage
  - 0.001 is about a reasonable strain under "working load conditions" (e.g. a building with a roof under normal conditions)
- *Engineering strain/stress* use the undeformed area and length; the *true strain/stress* use the deformed area/length; true strain is $\dfrac{\mathrm{d}}{\mathrm{d}x}$ of the displacement field
  - Engineering stress/strain is used instead of the true strain/stress because the latter is too hard to measure
- Using stress and strain allow us to compare structures of different sizes
  - e.g. a thicker cable will break at a higher load than a thinner cable, but if they're the same material, the stress that breaks them will be the same
- Stress and strain also apply for compression, as long as the compression is not too much and buckling doesn't happen

Young's Modulus and Relation to Hooke's Law

- Stress is proportional to strain by a constant E, the *Young's Modulus*: $\sigma = E\varepsilon$
  - Young's modulus is a property of the material and has the same dimensions as stress, typically MPa
  - The strains here are only as a result of carrying a load causing a stress; e.g. thermal or shrinkage strains should not be used here
  - Also known as the *material stiffness*
- k can be expressed in terms of a member's cross sectional area A, length L_0, and material stiffness E: $F = AE\dfrac{\Delta l}{L_0} = \dfrac{AE}{L_0}\Delta l \implies k = \dfrac{AE}{L_0}$
  - This shows that the axial stiffness is proportional to the cross-sectional area and material stiffness, and inversely proportional to length