Lecture 34, Dec 7, 2021

Reinforced Concrete Design Example

- What are the predicted: failure load f and failure mode (flexural? shear?) for the 2 beams tested at UofT?
- Beams:
  - YB2000/0
    - Simply supported, no shear reinforcement
    - Rectangular cross-section, 2000mm tall 300mm wide, $d = 1890\text{mm}$
    - 6 bars at the bottom (30M bars, $f_y = 457\text{MPa}$), 3 bars at the top (20M bars, $f_y = 433\text{MPa}$), no shear reinforcement
    - 5400mm from midpoint load P to supports
    - $f_c' = 36.6\text{MPa}, E_c = 28000\text{MPa}, \gamma = 24\text{kN/m}^3$
  - YB2000/4
    - Same dimensions
    - Vertical shear reinforcement, one leg of $A_v = 129\text{mm}^2$, $s = 590\text{mm}$, $f_{yt} = 468\text{MPa}$
- Analysis:
  1. Reactions, SFD, BMD: Consider both self-weight and applied load (same for both beams)
     - Span of 1080m, not considering the bit past the supports
     - Self-weight causes $w = 14.4\text{kN/m}$
     - Reaction forces due to self-weight are 77.8kN at each support
     - BMD due to self-weight is a parabola with peak $\dfrac{wL^2}{8} = 210\text{kN m}$
     - Point load causes reaction forces of $\dfrac{P}{2}$ (max shear), and max bending moment of $\dfrac{PL}{4} = 2.7P$
     - Combining the two SFDs and BMDs, the peak shear is at the support $V = \left(77.8 + \dfrac{P}{2}\right)\text{kN}$, and the peak moment $M = (210 + 2.7P)\,\text{kN m}$ is at the middle
  2. Bending strength check (same for both beams since the shear reinforcement does not affect bending)
     - No safety factors since we're trying to predict the failure load
     - $n = \dfrac{E_s}{E_c} = \dfrac{200000\text{MPa}}{28000\text{MPa}} = 7.14$
     - $\rho = \dfrac{A_s}{bd} = \dfrac{6 \cdot 700\text{mm}^2}{300\text{mm} \cdot 1890\text{mm}} = 0.00741 = 0.741\%$
     - $n\rho = 0.0529 \implies k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho = 0.277$ (depth of compression is 27.7% of d)
     - $jd = 1750\text{mm}$
     - $j = 1 - \dfrac{1}{3}k = 0.908$
       - The design code assumes $j = 0.9$
     - $f_s = \dfrac{M}{A_s jd} = f_y$ at bending failure due to yield (strain hardening ignored)
     - Steel yielding happens at $M = f_y A_s jd = 3292\text{kN m} = 210 + 2.7P \implies P = 1141\text{kN}$
     - Concrete crushing skipped since from experience it won't govern; on an exam note down the assumption if this is not being checked
  3. Shear strength
     - For YB2000/0:
       - $A_v = 0 \implies V_s = 0 \implies V_{ult} = V_c = \dfrac{230\sqrt{f_c'}}{1000 + 0.9d} b_w jd = \dfrac{230\sqrt{366}}{1000 + 0.9d} \cdot 300 \cdot 1750 = 265\text{kN}$
       - $265\text{kN} = 77.8 + \dfrac{P}{2} \implies P = 374\text{kN}$
       - Since this is smaller, it governs
       - The actual experiment failed at $P = 463\text{kN}$ (ratio of observed to predicted is 1.2)
     - For YB2000/4:
       - Check whether there is enough shear reinforcement: $\dfrac{A_v f_{yt}}{b_w s} \geq 0.06\sqrt{f_c'}$, for this beam this

doesn't quite work

- This beam was designed before the new rules, so we're going to assume there is enough shear reinforcement anyway (don't do this at work or on the exam!)

* $V_c = 0.18\sqrt{f'_c} b_w j d = 538\text{kN}$
* $V_s = \dfrac{A_v f_{yt} j}{s} \cot 35° = 250\text{kN}$
* $V_{ult} = V_c + V_s = 558 + 250 = 808\text{kN} = 77.8 + \dfrac{P}{2} \implies P = 1460\text{kN}$
* Since this is higher than the P from flexure, flexure governs for this beam
* Note that V_{max} was not checked because of time, but should be done!

- Note the significant increase in failure load by adding the shear reinforcement