Lecture 33, Dec 6, 2021

Reinforced Concrete Under Shear

- Shear causes diagonal tension, which could easily cause diagonal cracks to form
  - Bending cracks are vertical, shear cracks are diagonal
- Once concrete is cracked diagonally, there are two ways it can carry shear stresses:
  1. Aggregate interlock: The rough crack surfaces due to the aggregate (rock pieces) lock against each other, along with the longitudinal steel carry tension across the crack
  2. Shear reinforcement (aka stirrups or transverse reinforcement): Steel reinforcement bars perpendicular to the longitudinal reinforcement carry tension across the crack
     - It would be more effective to make these diagonal to be perpendicular to the tension direction, but doing so makes it much harder to construct
- Shear failures can be very aggressive and sudden so they're very dangerous (compared to yielding of longitudinal steel)

Shear Equation

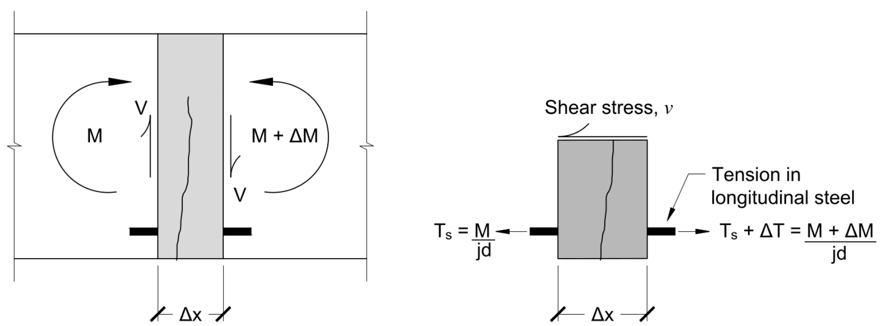

Figure 1: Derivation of the shear stresses in cracked reinforced concrete

- Jourawski's equation can no longer be used because the concrete's tensile capacity is severely reduced
- Recall that $M = T_s jd$ where jd is the flexural lever arm (vertical distance between the centroid of the flexural stresses and the centroid of the tensile stresses)
- Consider a longitudinal slice as shown above, then $M = T_s jd \implies V = \frac{\mathrm{d}M}{\mathrm{d}x} = \frac{\Delta T_s jd}{\Delta x} \implies \Delta T_s = \frac{V\Delta x}{jd}$
- Shear stress τ acts over an area with length Δx and width b_w (top right figure), so to satisfy equilibrium, $vb_w\Delta x = \Delta T_s = \frac{V\Delta x}{jd} \implies v = \frac{V}{b_w jd}$

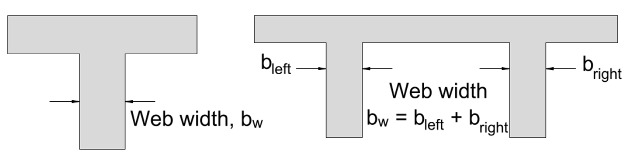

Figure 2: Effective web width

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  - Note that b_w may consider adjacent webs

- Shear failure happens when $v > v_c + v_s$, where v_c is the shear strength from aggregate interlock, and v_s is the shear strength from the steel shear reinforcement
  - $V_c = v_c b_w jd$ and $V_s = v_s b_w jd$ where the capital Vs are the shear forces
- If the member can't fail under shear tension, then the diagonal shear compression may also cause crushing of the concrete; the shear stress at which this occurs is defined as $v_{max} = 0.25f'_c$ by the Canadian concrete design code (f'_c is the compressive strength of concrete)
  - The failure shear force for concrete crushing is $0.25f'_c b_w jd$
  - We don't consider buckling in this case because typically the concrete is thick enough that buckling isn't an issue
- In summary the shear failure force V_r is equal to the sum of the concrete strength and steel strength, and this sum is less than V_{max}
  - To provide an adequate factor of safety steel strength terms are multiplied by 0.6 and concrete strength terms 0.5 (FoS of 1.67 in steel, 2 in concrete; this is because steel is manufactured under controlled conditions, while concrete is cast outdoors so has more variability)
  - $V_r = 0.5V_c + 0.6V_s \leq 0.5V_{max}$

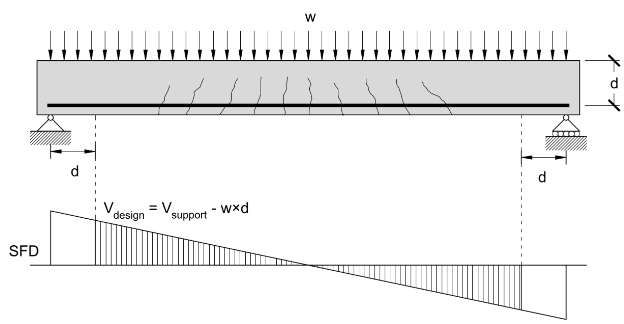

Figure 3: Location of shear force taken

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- Shear forces at reactions and point loads are typically not used because the added compression prevents the member from failing in shear tension at that location
  - Typically shear forces a distance d from the reaction or point load is used (d is the distance from the steel reinforcement and compression face, the same one used in flexural design)

Shear Capacity Without Shear Reinforcement

- Members without shear reinforcement rely solely on v_c (aggregate interlock) to carry shear once the concrete is cracked
- As the member depth increases, the cracks tend to get larger, which makes aggregate interlock less effective (referred to as the *size effect*)
- The shear strength of concrete without shear reinforcement is $v_c = \dfrac{230\sqrt{f'_c}}{1000 + 0.9d}$, so $V_c = v_c b_w jd = \dfrac{230\sqrt{f'_c}}{1000 + 0.9d} b_w jd$
  - Once again units have to be MPa and mm in this empirical equation

Shear Capacity With Shear Reinforcement

- Shear reinforcement bars are perpendicular to the longitudinal reinforcement and are commonly inserted by bending bars to form U shapes or hoops (*stirrups*)

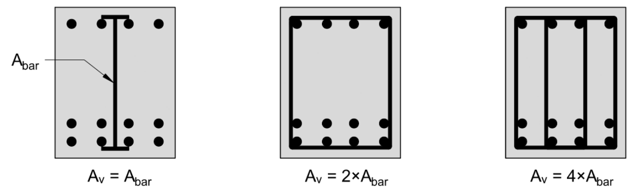

Figure 4: Types of shear reinforcement

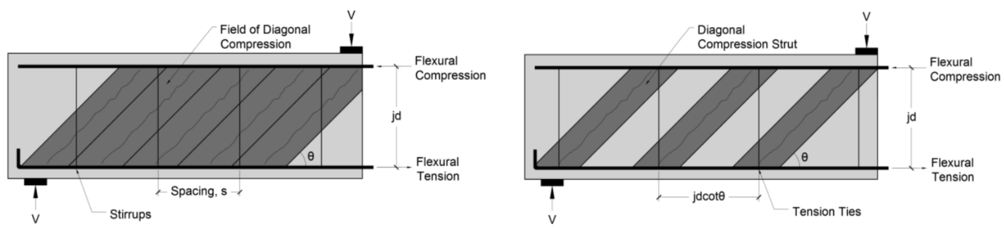

Figure 5: Diagonal stress fields and simplified truss model for reinforced concrete in shear

- The *area of shear reinforcement* A_v is the total cross-sectional area of the vertical bars
- Shear reinforcement provides shear strength V_s and controls the crack width, thus making aggregate interlock more effective and increases V_c as well
- The shear stress can be simplified into diagonal compression of angle θ and converted into a truss model, with height jd and shear reinforcement spaced s apart
  - The vertical tension members in this model are spaced $jd\cot\theta$ apart
  - $\dfrac{A}{jd\cot\theta} = \dfrac{A_v}{s} \implies A = \dfrac{A_v jd\cot\theta}{s}$
  - Failure occurs when the stress in these bars reach the yield stress, so $V_s = Af_y = \dfrac{f_y A_v jd}{s}\cot\theta$
  - The Canadian design code assumes diagonal stresses at $\theta = 35°$, thus $V_s = \dfrac{A_v f_y jd}{s}\cot 35°$
  - The shear strength attributed to the steel is $v_s = \dfrac{V_s}{b_w jd} = \dfrac{A_v f_y}{b_w s}\cot 35°$
- For small amounts of shear reinforcement V_c is unchanged, but if $\dfrac{A_v f_y}{b_w s} \geq 0.06\sqrt{f'_c}$, then $v_c = 0.18\sqrt{f'_c} \implies V_c = 0.18\sqrt{f'_c} b_w jd$
  - Note that if there's less than this amount of shear reinforcement, we essentially treat $V_s = 0$, because if there's not enough shear reinforcement then they may not cross the cracks so they might not work at all

Summary

- To evaluate the shear strength of a member:
  1. Obtain SFD, BMD, and determine max shear force V (at least d away from a reaction force or point load), calculate k, j
  2. Check the amount of shear reinforcement to determine the right equation to use: $V_c = \begin{cases} \dfrac{230\sqrt{f'_c}}{1000 + 0.9d} b_w jd & \dfrac{A_v f_y}{b_w s} < 0.06\sqrt{f'_c} \\ 0.18\sqrt{f'_c} b_w jd & \text{otherwise} \end{cases}$
  3. If there is shear reinforcement, $V_s = \dfrac{A_v f_y jd}{s}\cot 35°$
  4. Calculate the shear strength: $V_r = V_c + V_s \leq V_{max} = 0.25 f'_c b_w jd$
  5. Check for failure, when $V = V_r$
- To design shear reinforcement:

1. Obtain SFD, BMD, max V, k, j
2. Check if $V \geq 0.5V_{max} = 0.5 \cdot 0.25 f_c' b_w jd$; if so then the cross-section is too small and needs to be resized
3. Check whether $V < 0.5V_c = 0.5\frac{230\sqrt{f_c'}}{1000 + 0.9d} b_w jd$; if so then aggregate interlock alone can handle the shear, so the design is complete
4. Provide the minimum amount of shear reinforcement, $\frac{A_v f_w}{b_w s} = 0.06\sqrt{f_c'} \implies s = \frac{A_v f_y}{0.06\sqrt{f_c'} b_w}$, and then check whether $V < V_r = 0.5V_c + 0.6V_s = 0.5 \cdot 0.18\sqrt{f_c'} b_w jd + 0.6 \cdot \frac{A_v f_y jd}{s} \cot 35°$
5. If the shear capacity is still too low, then determine the minimum s needed to make $V = V_r$: $s = \frac{0.6 \cdot A_v f_y jd \cot 35°}{V - 0.5 \cdot 0.18\sqrt{f_c'} b_w jd}$