Lecture 3, Sep 15, 2021

Types of Bridges

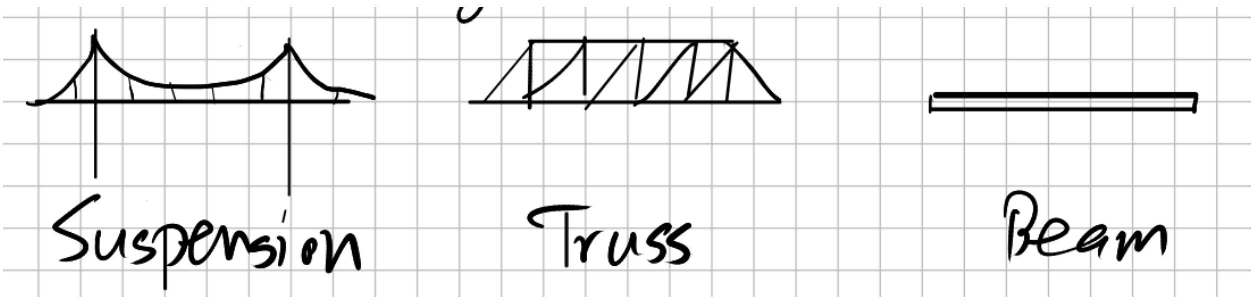

Figure 1: Bridge types

- Suspension bridges consist of cables held up by towers, with the weight of the bridge being connected to the main cable using vertical cables
- The weight of the main bridge is transferred to the cable, which is then transferred to the towers and then the ground

Internal Forces

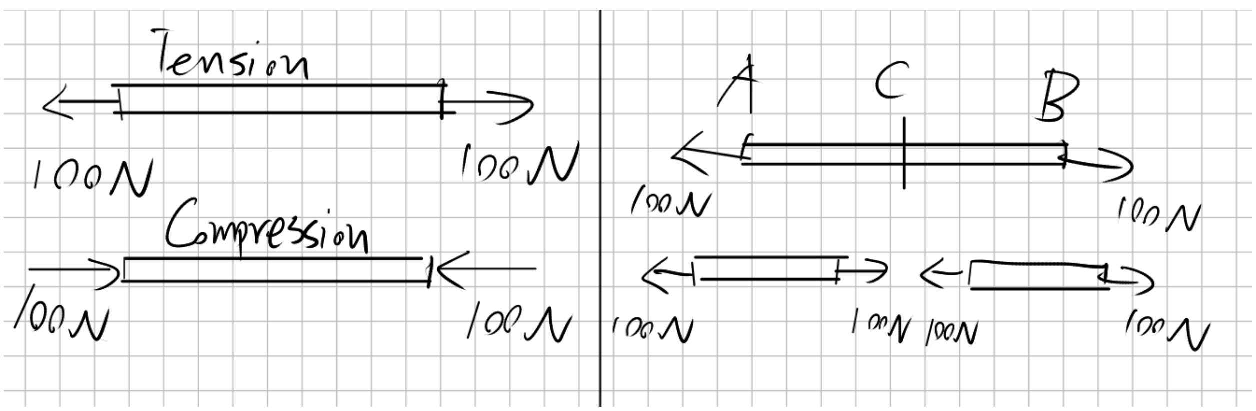

Figure 2: Internal forces

- Tension forces pull on both sides of an element, and tries to make it longer
  - If each end is being pulled with a 100N force, the tension is still 100N, not 200
  - The element "transmits" the force from one end to the other end
- Compression forces push in on the element, trying to make it shorter
- Key idea: **If the entire structure is in equilibrium, all its constituent parts are also in equilibrium.**
  - Example: If a beam is being pulled on each end with 100N and thus in equilibrium, we can break it apart at an arbitrary point, and expose an internal force; the internal forces will also be 100N and balance out the forces at the ends; this is the reason why tension is 100 and not 200

Cable Structure

- When a cable is attached at the ends and only supporting its own weight, it takes the shape of a **catenary** (modelled by $y = \cosh(e^{\frac{x}{a}} + e^{-\frac{x}{a}}) + b$, a = span, b = vertical offset, not a parabola)
  - This is a different shape from uniform loading because the load per unit length of cable is constant, not load per unit length of the span; so at the ends, there is more load per x distance

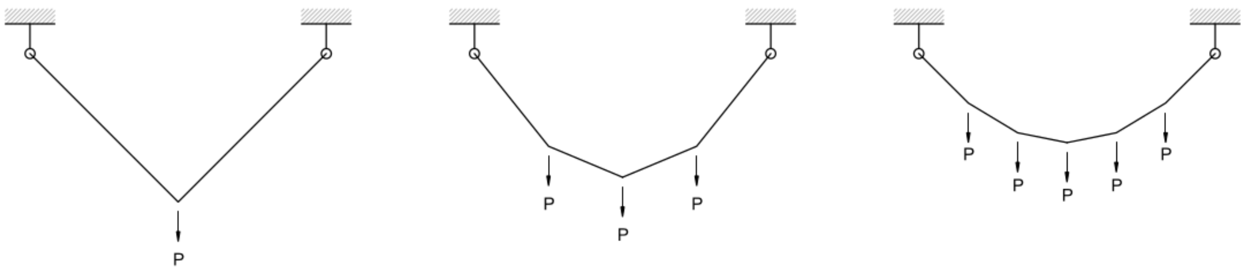

Figure 3: Cable shape

- If we add loads much heavier than the weight of the cable itself onto the cable, then the effect of the cable's weight will be negligible and the cable becomes **piecewise linear**
- Typically in a suspension bridge the horizontal spacing between loads is constant (not the cable length between loads)
- When the weights are uniformly loaded, the cable takes the shape of a *parabola*

Forces

- Consider a suspension bridge uniformly loaded with 7 masses each with weight P

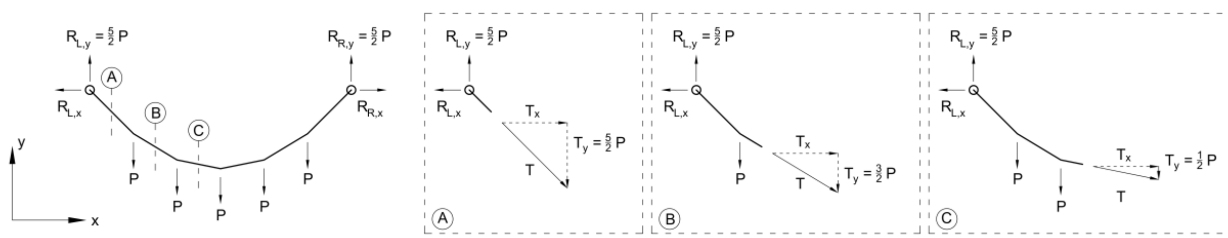

Figure 4: Force calculations

- At the endpoints, the vertical component of force is equal to half of the total weight of the bridge due to symmetry
- In each segment, the horizontal component of tension remains the same and is equal to the tension at the ends
- In the 2 end segments, the vertical component of force is equal to the endpoints; then as you move in to the middle of the bridge, each segment has vertical tension reduced by P, with the middle segments having tension $\frac{1}{2}P$