Lecture 25, Nov 15, 2021

Shear Stresses

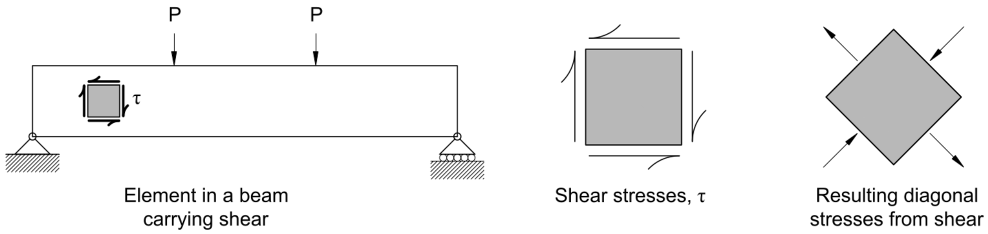

Figure 1: Shear stresses in beams

- Shear stress, denoted by τ, like axial stress, is also equal to the force over an area: $\tau = \dfrac{V}{A}$
  - Force here is the shear force
  - Area here is related, but not equal to, the cross sectional area of the beam
- In the figure above, a section is cut from a beam; there are vertical shear stresses on the left and right sides, which satisfy vertical equilibrium, but since they form a couple and produce a moment, *complementary shear stresses* exist on the top and bottom faces to satisfy rotational equilibrium
- These shear stresses and complimentary shear stresses can be resolved into diagonal tension and compression
  - Shear stresses can be transformed into diagonal axial stresses and back again, but for beam design the shear stresses can be used directly
- While axial stresses tend to cause the material to extend or contract and change volume, shear stresses tend to change the shape of the material while maintaining its volume
  - Shear stresses are usually denoted γ
  - Axial stresses come from forces perpendicular to the cross section and changes squares to rectangles (deforms and gets longer)
  - Shear stresses come from forces parallel to the cross section and deform a square into a parallelogram
  - For a linear elastic material, there are no axial stresses produced by shear

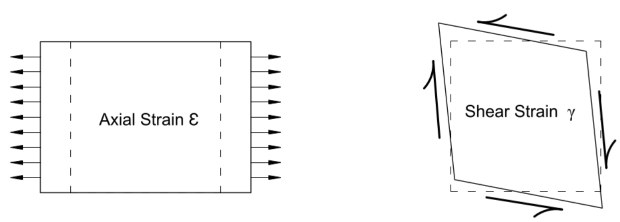

Figure 2: Axial deformation vs shear deformation

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- Shear stresses cause different modes of failure; in wooden members, adjacent elements can slide past each other; in more brittle materials such as concrete diagonal cracking may occur; finally diagonal compressive shear stresses may lead to diagonal buckling
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- If we look at a small section cut in the middle of the beam, the shear stresses on the top and bottom of the cut are always the same as the left and right sides

Figure 3: Failure due to shear stresses

- Horizontal shear stresses always comes with vertical shear stresses
- Shear stresses are 0 at the top and bottom of the beam

Shear Deformations

- Consider a square section, with 4 targets marked at the 4 corners; after this section is deformed by shear, the distance between one pair of diagonals got logger and the other distance between the other pair of diagonals got shorter
- Inside shear is both tension and compression, so it causes materials to elongate and contract at the same time
- Notice the way the concrete beam failed in the image above; the diagonal failure is because there is diagonal tension

Jourawski's (Zhuravskii's) Equation

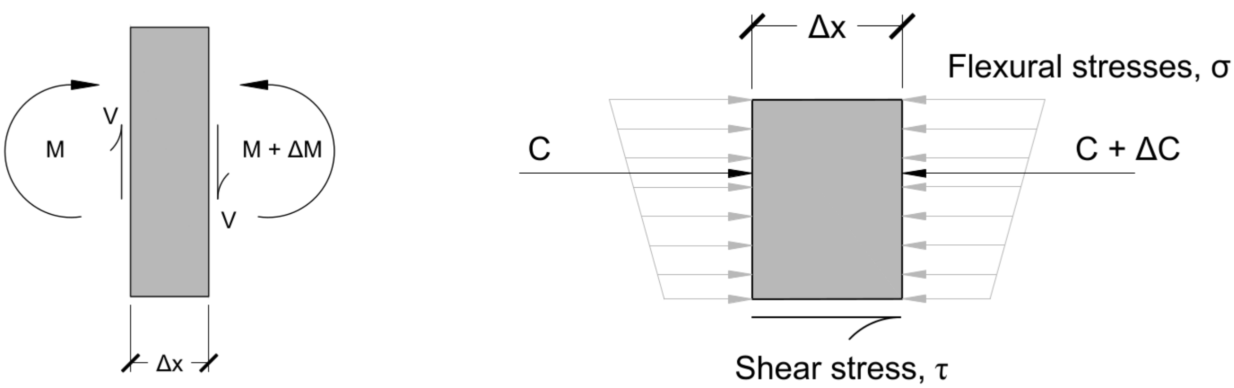

Figure 4: Jourawski's Equation derivation

- Since shear force is the derivative of bending moment, where there is shear force, there will be a change in bending moment
- Consider a beam with width b and second moment of area I, with a cut Δx units wide in the diagram above; the right side has a higher bending of $M + \Delta M$
- In the right diagram, the cut is further cut at some depth; the higher moment on the right side creates higher flexural stresses and causes a net horizontal force of ΔC to the left

- If we cut it at a depth of y_0, $\Delta C = \int_{y_0}^{y_{top}} \sigma(y)\,\mathrm{d}y = \int_{y_0}^{y_{top}} \frac{\Delta M y}{I}\,\mathrm{d}y = \frac{\Delta M}{I}\int_{y_0}^{y_{top}} y\,\mathrm{d}y = \frac{\Delta M}{I}Q$
    - Q is the quantity $\int_{y_0}^{y_{top}} y\,\mathrm{d}y$, which is a first moment of area about the centroidal axis of the membe
- For this member to be in equilibrium, there must be a shear force on its underside also with magnitude ΔC; since $\tau = \frac{V}{A} \implies V = \tau A = \tau b \Delta x$
    - The area over which the shear stress acts is the beam's width times the length of the cut
- Therefore, $\frac{\Delta M}{I}Q = \Delta C = \tau b \Delta x \implies \tau = \frac{\Delta M Q}{I b \Delta x} = \tau \frac{Q}{IB} \cdot \frac{\Delta M}{\Delta x}$, and as $\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \frac{\mathrm{d}M}{\mathrm{d}x} = V$, we have $\tau = \frac{VQ}{Ib}$
    - This equation lets us calculate the shear stress and is called *Jourawski's Equation*
- $\tau = \frac{VQ}{Ib}$ computes the horizontal shear force, but since the vertical shear forces are always the same it also computes the vertical shear forces

Calculating Q

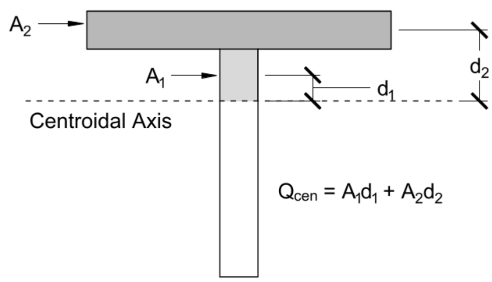

Figure 5: Calculating Q for a more complex region

- Recall that $\bar{y} = \frac{\int_A y\,\mathrm{d}A}{\int_A \mathrm{d}A} \implies \int_A y\,\mathrm{d}A = \bar{y}\int_A \mathrm{d}A = \bar{y}A$
- Therefore we can calculate Q for a certain depth by first finding A, the area of the cross section above the depth of interest, then determine the distance between the centroidal axis of the section of interest and the centroidal axis, d, and using $Q = Ad$
- For more complex regions we can break it up into simpler components and use $Q = \int_{i=1}^{n} A_i d_i$
- We don't care about when Q is negative since the direction of the shear stress doesn't matter, so Q is always an absolute value
- The integral for Q can be evaluated from the top of the cross section or the bottom: $\int_{y_{bot}}^{y_0} y\,\mathrm{d}A = \int_{y_0}^{y_{top}} y\,\mathrm{d}A$
    - This is because $\int_{y_{bot}}^{y_0} y\,\mathrm{d}A + \int_{y_0}^{y_{top}} y\,\mathrm{d}A = \int_{y_{bot}}^{y_{top}} y\,\mathrm{d}A$, but the first moment of area about the centroidal axis is always zero
    - Therefore $\int_{y_{bot}}^{y_0} y\,\mathrm{d}A = -\int_{y_0}^{y_{top}} y\,\mathrm{d}A$, but because we don't care about the sign of Q, we can say that they're equal

Distribution of Shear Stresses

- Like flexural stresses, shear stresses are nonconstant over the cross section since Q depends on the depth that the shear force is being calculated
- In general, Q has the following properties:
  1. Q varies parabolically over the height of the member
  2. Q and thus τ is zero at the very top and bottom of the member
  3. Q is maximized at the centroidal axis of the member
- If we integrated the shear stresses across y we will get the shear force V