Lecture 20, Oct 26, 2021

Stress Resultants: Beyond Tension and Compression

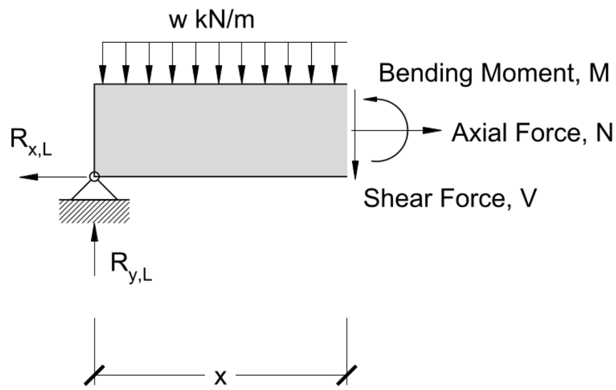

Figure 1: Stress resultants

- By cutting a beam we expose internal forces; these forces are the *stress resultants*
- Note that the overall beam is in neither tension nor compression
- The horizontal force parallel to the longitudinal axis is the *axial load* N or P, the vertical force perpendicular to the axis is the *shear force* V and the moment is the *bending moment* M
  - If there is too much deformation due to the shear force, the beam will deform in a shear transform
  - We design our trusses so that both the moment and shear force are zero; in reality there is some caused by the self-weight but this is ignored
- Using the equations of equilibrium: $\begin{cases} \sum F_x = 0 & \implies N = R_{x,L} \\ \sum F_y = 0 & \implies V = R_{y,L} - wx \\ \sum M_o = 0 & \implies M = R_{y,L}x - (wx)\left(\frac{1}{2}x\right) \end{cases}$
- Notice that unlike tension/compression these forces change depending on where you cut the beam, i.e. they vary over the length

Shear Force and Bending Moment Diagrams

- It turns out that the shear force is related to the vertical applied loads $w(x)$ by $w(x) = \frac{\mathrm{d}}{\mathrm{d}x}V(x)$, and the moment is related $\frac{\mathrm{d}M}{\mathrm{d}x} = V(x)$; i.e. the shear is the derivative of bending moment, and the applied load is the derivative of shear
  - Applied loads will increase the shear; the shear force has sudden jumps at locations where there are concentrated reaction forces or loads
  - Shear forces will increase the moment
- $\Delta M = \int_A^B V(x)\,\mathrm{d}x$ and $\Delta V = \int_A^B w(x)\,\mathrm{d}x$
- Therefore we can find the graph of shear and moment over the length of a member by integrating the loads
- For a shear force diagram, imagine you're walking across the bridge from the left to the right and accumulating the loads
- For a bending moment diagram, take the area under the shear force diagram
- We know the shear force and bending moment diagrams are correct if they start and end at 0

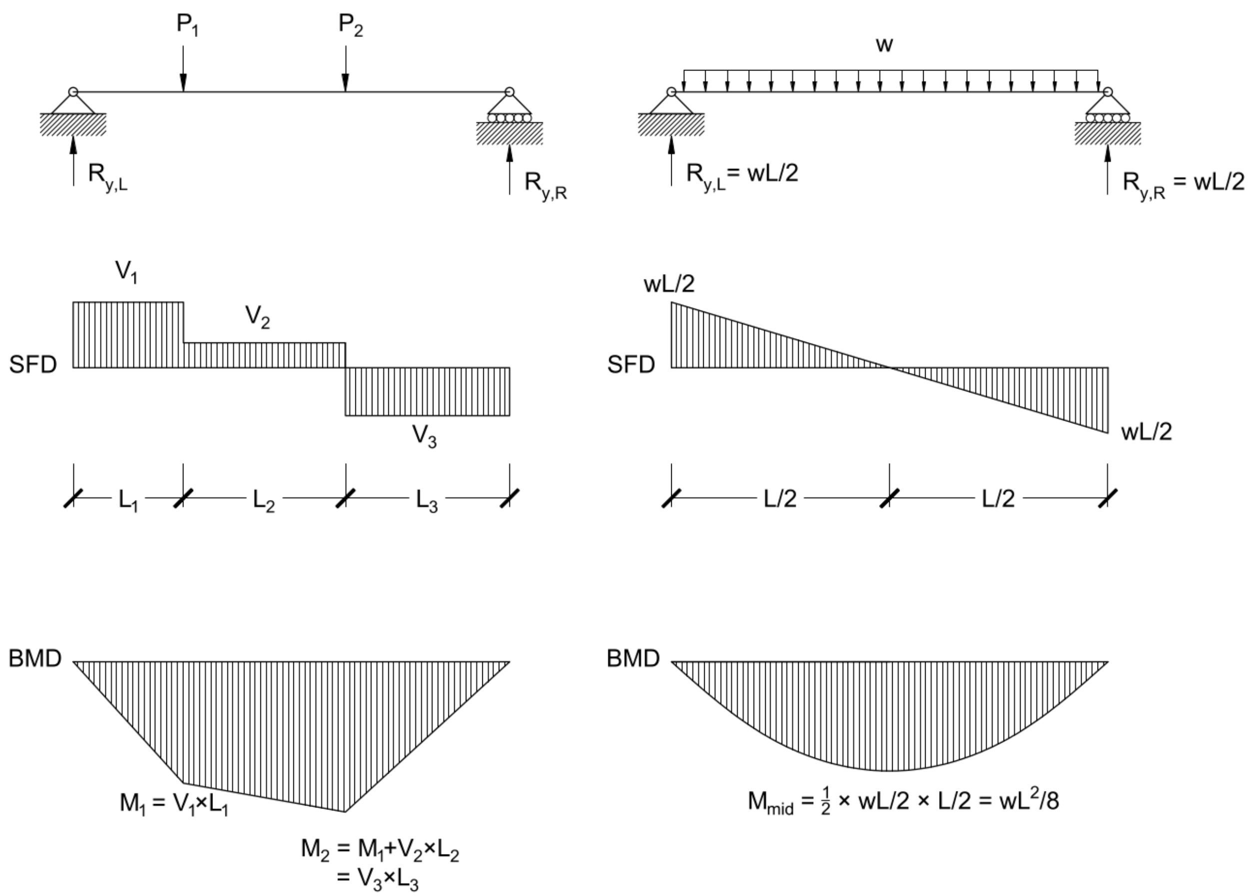

Figure 2: Shear force and bending moment diagram

- Upward loads cause positive shear and downward loads cause negative shear (from left to right)
- Positive bending moments are drawn below the axis and causes a positive moment (from the left to the right); negative moments are drawn above the axis and cause negative moments

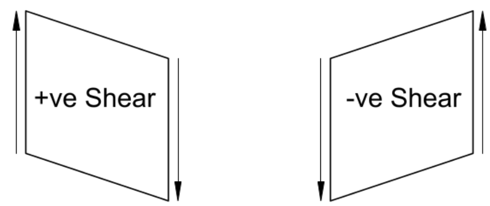

Figure 3: Sign convention for shear force

-

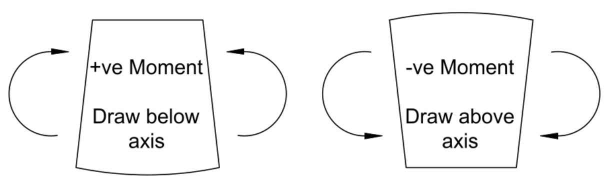

Figure 4: Sign convention for bending moment

-