Lecture 11, Oct 5, 2021

Structural Analysis

- Often we want to analyze the forces in a structure in response to loads such as objects, wind, earthquakes, etc
- *Stick models* are used as simplifications, with *members* connected at *joints*
- Member types:
  - Mostly horizontal: Beams (usually with less axial compression and maybe more bending)
  - Mostly vertical: Columns (usually with a lot of axial compression)
  - ~~Diagonal: Beam-Columns~~
- Types of loads:
  - Axial loads $F_x = N$ (normal to cross section)
  - Shear loads $F_y = V$ (vertical)
  - Moments and internal bending moments M

Supports

Table 11.1 – Types of supports and their reaction forces

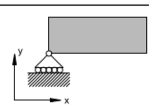
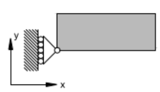
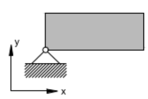
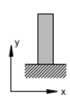
Name	Symbol	Permitted Degrees of Freedom	Restrained Degrees of Freedom	Support Reactions
Roller		$\Delta x, \theta_{xy}$	$\Delta y = 0$	F_y
		$\Delta y, \theta_{xy}$	$\Delta x = 0$	F_x
Pin		θ_{xy}	$\Delta x = \Delta y = 0$	F_x, F_y
Fixed end		None	$\Delta x = \Delta y = \theta_{xy} = 0$	F_x, F_y, M_{xy}

Figure 1: Types of supports

- Supports hold up the structure and create *reaction forces* to hold up the structure
- The reaction forces are closely related to the level of restraint of the support; e.g. a hinge stops all translational movement but allows rotational movement
- As the amount of restraint provided by the support increases, its ability to provide a reaction force along that degree of freedom increases
- In structural engineering, the 3 most common types of supports and connections are *rollers*, *pins*, and *fixed ends*:
  1. Rollers: Only resist translational forces in 1 direction and cannot resist forces in other directions **or moments**
  2. Pins/hinges: Cannot resist moment but resist translational movement
     - When deformed, angles may change
     - When cut for FBD, there is an internal horizontal force and vertical force that are equal and opposite between diagrams
  3. Fixed ends/rigid connections: Allows no movement
     - Even when deformed, fixed ends have angles of 90°, and the beams just bend

  - When cut for FBD, there is an internal horizontal force, vertical force, and moment that are equal and opposite between diagrams
- Whenever there is no motion in a degree of freedom for a support, there is a reaction force along that degree of freedom
- Note: Supports in real life cannot be perfectly described by the 3 idealized supports above, so choosing the type that best represents real life requires engineering judgement and experience
- A *degree of freedom* in this context is how many variables are needed to describe the system
  - 3 degrees of freedom, 2 translational, 1 rotational, are used to describe a non-deformable body in 2D
  - Supports fix some of them but leave some of them free, reducing the number of degrees of freedom

Solving for Reaction Forces – Free Body Diagrams

- To solve for reaction forces in a structure, free body diagrams are drawn for every interaction
- The 3 key equilibrium equations are used to solve for the unknown forces: $\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$
  - Note: In 3 dimensions there are 3 axes of translational equilibrium and also 3 axes of rotational equilibrium $\theta_x, \theta_y, \theta_z$, together 6 degrees of freedom

Statically Determinate Structures

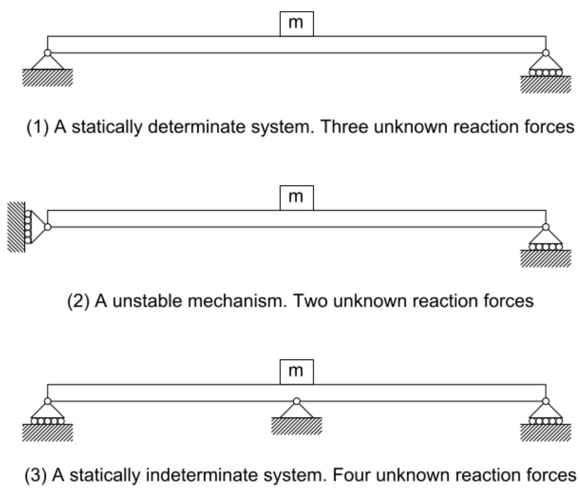
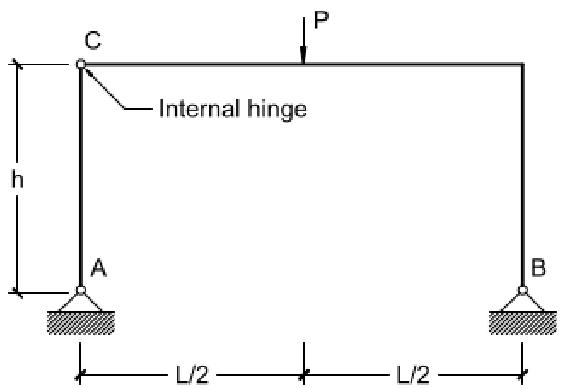


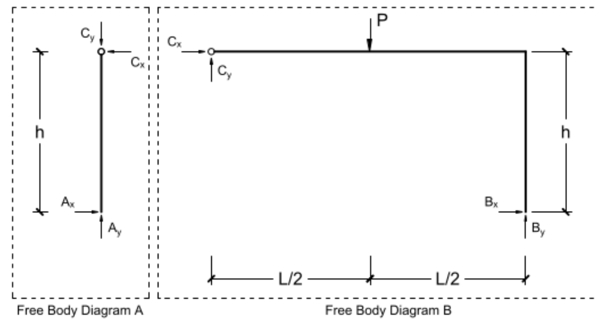
Figure 2: Examples of the different types of systems

- A structure is *statically determinate* if its reaction forces can be directly solved for using the 3 equilibrium equations
- Forces in statically determinate structures do not depend on the stiffness of the structure
- Most simple 2D structures are statically determinate if their supports provide 3 reaction forces (since there are 3 equilibrium equations)
- Structures with fewer reaction forces than the number of equilibrium equations are called *mechanisms*, because they are *unstable* and can accelerate when subject to an applied load
  - If the system ends up being unsolvable the system would be moving since one of the degrees of freedom is not in equilibrium

  - If the system ends up being solvable, then it is not moving, but any applied force in the right direction would cause it to move
- *Statically indeterminate* structures have more reaction forces than the number of equilibrium equations, so solving them require considering other factors such as stiffness and load distribution
  - The *degree of indeterminacy* is the number of reaction forces minus the number of equilibrium equations
- (1) is a *simply supported beam* (pin on one end, roller on the other end) – these beams can bend and change length freely

Example: Internal Hinges





- Some structures are built with an internal hinge, which provide only two translational forces
- We can split the FBD at the hinge, which introduces 2 internal hinge forces, but 3 additional equilibrium equations, effectively reducing the indeterminacy by 1
- On the left: $\begin{cases} \sum F_x = 0 & \implies A_x - C_x = 0 \\ \sum F_y = 0 & \implies A_y - C_y = 0 \\ \sum M = 0 & \implies hC_x = 0 \end{cases}$
- On the right: $\begin{cases} \sum F_x = 0 & \implies B_x + C_x = 0 \\ \sum F_y = 0 & \implies B_y + C_y - P = 0 \\ \sum M = 0 & \implies \dfrac{L}{2}P - hC_x - LC_y = 0 \end{cases}$
- Since we have 6 equations and 6 unknowns this system is now statically determinate